INTERNAL FINITE SIZE SOURCE IN A HOMOGENEOUS HALF-SPACE

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Utilization of nondestructive control methods for construction and materials is associated with the construction and decoding of wave fields produced by inclusions and inhomogeneities of different kinds in a medium. The modeling of seismic foci also reduces to similar problems [1]. The problem of the emission by a cylinder of finite size and radius r_0 into an infinite medium was apparently first investigated in [2]. In this paper the cylinder walls were subjected to nonstationary pressures (longitudinal and two kinds of shearing). With such a kind of loads taken into account, representations are obtained for the amplitudes of longitudinal and transverse polarized waves. Moreover, the question of the energy distribution between the kinds of waves mentioned is studied. It is shown that under the effect of just the longitudinal pressure, an SV-wave emerges from the source, whose amplitude is 1.6 times greater than the P-wave and which is directed at a 45° angle. Recently the main emphasis has been all the more on taking account of reflections from the surface and inner inhomogeneities (see [1, 3, 4], say) rather than on studying the emitted field (problem for an infinite medium).

The problem occurring in a study of wave fields excited in an elastic inhomogeneous half-space by a deep infinitely thin source of finite size, the pile foundation of a vibration installation, say, is considered in this paper. The solution of the problem is a superposition of the wave field emitted directly by the source and reflected from the half-space surface. The vertical and horizontal vibrations of the pile are simulated by a bulk force distributed under the surface of an elastic half-space along a finite segment. Analytic representations are obtained for the amplitudes of the longitudinal, transverse and Rayleigh waves in the far zone from the source.

1. A homogeneous elastic half-space $(-\infty \leqslant x,\ y \leqslant \infty,\ -\infty \leqslant z \leqslant 0)$ is considered. Steady harmonic vibrations of the medium $\mathbf{v} = \mathrm{Re}[\mathbf{u}e^{i\omega t}]$ excited by an infinitely thin deep source of finite length are described by the Lamé dynamic equations

$$(\lambda + 2\mu)\mathrm{grad}\ \mathrm{div}\ \mathbf{u} - \mu\ \mathrm{rot}\ \mathrm{rot}\ \mathbf{u} + \mathbf{f} + \rho\omega^2\mathbf{u} = 0 \qquad (1.1)$$

with the boundary conditions

$$\tau_{xz} = \tau_{yz} = \sigma_{zz} = 0,\ -\infty \leqslant x,\ y \leqslant \infty,\ z = 0. \qquad (1.2)$$

The displacements $\mathbf{u} = \{u(x, y, z), v(x, y, z), w(x, y, z)\}$ should vanish at infinity and the radiation condition resulting from the principle of limit absorption [5] should be satisfied. In the relationships (1.1) ω is the cyclic frequency of source vibration, λ, μ are Lamé coefficients, ρ is density of the medium, $\mathbf{f} = \{f_1, f_2, f_3\}$ is the bulk force vector simulating the action of a vertically oriented source on the medium, $f_l(x, y, z) = f_l(z)\,\delta(x - x_m, y - y_m)$, $-h \leqslant z \leqslant -h_0$. All the physical quantities are reduced to dimensionless form. The Lamé coefficients are referred to a certain characteristic value of the shear modulus of the medium $\mu_0 = 10^9$ N/m^2 the density ρ to the density $\rho_0 = 10^3$ kg/m^3, and the linear dimensions to a characteristic linear dimension $\ell_0 = 1$ m. In this case the generalized frequency $\omega = 2\pi\nu l_0\sqrt{\rho_0/\mu_0} = 6\cdot 10^{-3}\nu$, where ν is the frequency in Hz.

After application of the Fourier transformation with parameters α_1, α_2 in the coordinates x and y the initial boundary value problem is reduced to the solution of systems of ordinary differential equations

$$\mathbf{Y}' = A\mathbf{Y} + \mathbf{H}; \qquad (1.3)$$

$$T\mathbf{Y}|_{z=0} = 0,\ \mathbf{Y} \to 0,\ z \to -\infty; \qquad (1.4)$$

$$\mathbf{X}' = B\mathbf{X} + \mathbf{P}; \qquad (1.5)$$

$$(\mathbf{L}\cdot\mathbf{X})|_{z=0} = 0,\ \mathbf{X} \to 0,\quad z \to -\infty. \qquad (1.6)$$

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Here A, B, and T are matrices with the constant coefficients

$$A=\begin{pmatrix}0 & 1 & 0 & 0\\ a_{21} & 0 & 0 & a_{24}\\ 0 & 0 & 0 & 1\\ 0 & a_{42} & a_{43} & 0\end{pmatrix},\quad B=\begin{pmatrix}0 & 1\\ b_{21} & 0\end{pmatrix},$$

$$T=\begin{pmatrix}-\lambda\alpha^2 & 0 & 0 & \lambda+2\mu\\ 0 & -i\mu\alpha^2 & -i\mu\alpha^2 & 0\end{pmatrix},$$

$$b_{21}=(\mu\alpha^2-\rho\omega^2)/\mu,\ a_{21}=[\alpha^2(\lambda+2\mu)-\rho\omega^2]/\mu,$$
$$a_{24}=-(\lambda+\mu)/\mu,\ a_{42}=\alpha^2(\lambda+\mu)/(\lambda+2\mu),$$
$$a_{43}=(\alpha^2\mu-\rho\omega^2)/(\lambda+2\mu),\ \alpha^2=\alpha_1^2+\alpha_2^2.$$

The prime denotes differentiation with respect to z and the vectors $\mathbf{Y},\mathbf{X},\mathbf{H},\mathbf{P},\mathbf{L}$ have the form

$$\mathbf{Y}=\{\Phi,\ \Phi',\ W,\ W'\},\ \mathbf{X}=\{\Psi,\ \Psi'\},$$
$$\mathbf{H}=\left\{0,\ -\frac{i}{\mu\alpha^2}(\alpha_1F_1+\alpha_2F_2),\ 0,\ -\frac{1}{\lambda+2\mu}F_3\right\},$$
$$\mathbf{P}=\left\{0,\ -\frac{i}{\mu\alpha^2}(\alpha_2F_1-\alpha_1F_2)\right\},\quad \mathbf{L}=\{0,\ -i\mu\alpha^2\},$$

where $F_l=f_l(z)\,e^{i(\alpha_1x_m+\alpha_2y_m)}$, $\Phi,\ \Psi,\ W(\alpha_1,\alpha_2,z)=\int\limits_{-\infty}^{+\infty}\int\varphi,\ \psi,\ w(x,y,z)\ e^{i(\alpha_1x+\alpha_2y)}dxdy$ and the functions $\varphi(x,y,z)$ and $\psi(x,y,z)$ are connected with the displacements $\mathbf{u}=\{u,v,w\}$ by the following relationships $u=\partial\varphi/\partial x+\partial\psi/\partial y$, $v=\partial\varphi/\partial y-\partial\psi/\partial x$.

The solution of the inhomogeneous systems (1.3) and (1.5) is sought in the form of a sum of the particular solution determined by the method of variation of constants and a general solution of the corresponding homogeneous system

$$\mathbf{Y}=\sum_{k=1}^{4}d_ke^{\gamma_kz}\mathbf{m}_k+\sum_{k=1}^{4}t_k(z)\,e^{\gamma_kz}\mathbf{m}_k; \tag{1.7}$$

$$\mathbf{X}=\sum_{k=1}^{2}c_ke^{s_kz}\mathbf{n}_k+\sum_{k=1}^{2}\tau_k(z)\,e^{s_kz}\mathbf{n}_k. \tag{1.8}$$

Here $\gamma_{1,2}=\pm\sigma_1=\pm\sqrt{\alpha^2-\varkappa_1^2}$; $\gamma_{3,4}=\pm\sigma_2=\pm\sqrt{\alpha^2-\varkappa_2^2}$; $\varkappa_1^2=\frac{\rho\omega^2}{\lambda+2\mu}$; $\varkappa_2^2=\frac{\rho\omega^2}{\mu}$; $s_k=\pm\sigma_2$; $\mathbf{m}_k$, $\mathbf{n}_k$ are the eigenvalues and eigenvectors of the matrices A and B. The single-valued branch of the radicals σ_1, σ_2 is determined by the condition $\mathrm{Re}\,\sigma\geqslant 0, \mathrm{Im}\,\sigma\leqslant 0$. Such a selection of the branch satisfies the radiation principle [5]. The coefficients $t_k(z)$, $\tau_k(z)$ have the form

$$t_k(z)=q_1\left(\pm\frac{i\alpha_1}{\sigma_1}I_{11}^{\mp}(z)\pm\frac{i\alpha_2}{\sigma_1}I_{21}^{\mp}(z)-I_{31}^{\mp}(z)\right),$$
$$t_{k+2}(z)=q_1\left(-\frac{i\alpha_1}{\alpha^2}I_{12}^{\mp}(z)-\frac{i\alpha_2}{\alpha^2}I_{22}^{\mp}(z)\pm\frac{1}{\sigma_2}I_{32}^{\mp}(z)\right),$$
$$\tau_k(z)=\pm q_2\left(\alpha_2I_{12}^{\mp}(z)-\alpha_1I_{22}^{\mp}(z)\right),$$
$$I_{ln}^{\mp}(z)=\int\limits_0^z f_l(\xi)\,e^{\mp\sigma_n\xi}d\xi,$$
$$q_1=e^{i(\alpha_1x_m+\alpha_2y_m)}/2\rho\omega^2,$$
$$q_2=e^{i(\alpha_1x_m+\alpha_2y_m)}/2\mu\sigma_2\alpha^2,$$

where the upper sign corresponds to k = 1 and the lower to k = 2. The unknown coefficients d_k and c_k of the solutions of the homogeneous systems are determined from the boundary conditions for z = 0 and at infinity. To satisfy the condition as $z\to-\infty$ it is necessary to set

$$d_2=-t_2(-h),\ d_4=-t_4(-h),\ c_2=-\tau_2(-h) \tag{1.9}$$

in the solutions (1.7) and (1.8). The coefficients $t_2(-h)$, $t_4(-h)$, $\tau_2(-h)$ differ from $t_2(z)$, $t_4(z)$, $\tau_2(z)$ by the fact that in the expressions determining them must be used $I_{ln}^{\mp}(-h)=\int\limits_{-h_0}^{-h}\times f_l(\xi)\,e^{\mp\sigma_n\xi}d\xi$ in place of $I_{ln}^{\mp}(z)$. Selection of the coefficients d_2, d_4, c_2 in the form (1.9) is justified by the fact that by solving the two homogeneous problems of the type (1.3) and (1.5) for the domains $-h_0 \leqslant z \leqslant 0$, $-\infty \leqslant z \leqslant -h$ and one inhomogeneous problem for the domain $-h \leqslant z \leqslant -h_0$, and then merging the solutions on the basis of the condition of equality of the displacements and stresses on the domain boundaries, we arrive at the relationships (1.9). The remaining unknown coefficients d_1, d_3 are determined from the conditions (1.4) and (1.6) for z = 0.

Finding the solution of the boundary value problems (1.3)-(1.6) and applying the inverse Fourier transform to the equalities

$$U(\alpha_1, \alpha_2, z) = -i\alpha_1\Phi(\alpha_1, \alpha_2, z) - i\alpha_2\Psi(\alpha_1, \alpha_2, z),$$
$$V(\alpha_1, \alpha_2, z) = -i\alpha_2\Phi(\alpha_1, \alpha_2, z) + i\alpha_1\Psi(\alpha_1, \alpha_2, z),$$
$$W(\alpha_1, \alpha_2, z) = W(\alpha_1, \alpha_2, z),$$

we obtain the solution of the initial problem (1.1) and (1.2)

$$\mathbf{u}(x, y, z) = \frac{1}{4\pi^2}\int_{\Gamma_1}\int_{\Gamma_2}\sum_{k=1}^{2}\left(\mathrm{Q}_k e^{\sigma_k z} + \mathrm{G}_k e^{-\sigma_k z}\right) e^{-i[\alpha_1(x-x_m)+\alpha_2(y-y_m)]}\, d\alpha_1 d\alpha_2,$$

$$\mathrm{Q}_k = \begin{pmatrix} -i\alpha_1 & 0 \\ -i\alpha_2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} J_k^1 - (D_1P_k + D_2M_k) \\ J_k^2 - (D_1R_k + D_2S_k) \end{pmatrix} + (k-1)\begin{pmatrix} -i\alpha_2 \\ i\alpha_1 \\ 0 \end{pmatrix}(\tau_1(z) - D_3N),$$

$$\mathrm{G}_k = \begin{pmatrix} -i\alpha_1 & 0 \\ -i\alpha_2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} H_k^1 \\ H_k^2 \end{pmatrix} + (k-1)\begin{pmatrix} -i\alpha_2 \\ i\alpha_1 \\ 0 \end{pmatrix}(\tau_2(z) - \tau_2(-h)), \qquad (1.10)$$

$$D_k = -t_2(-h)\,\{T\mathrm{m}_2\}_k - t_4(-h)\,\{T\mathrm{m}_4\}_k,\quad k = 1, 2,$$
$$D_3 = -\tau_2(-h)\, i\mu\alpha^2\sigma_2,$$
$$J_k^p = t_{2k-1}(z)\,\mathrm{m}_{2k-1}^{2p-1},\quad p, k = 1, 2,$$
$$H_k^p = [t_{2k}(z) - t_{2k}(-h)]\,\mathrm{m}_{2k}^{2p-1},\quad p, k = 1, 2.$$

The explicit form of the functions M, N, P, R, S is presented in [5], and the contours of integration are selected in conformity with the limit absorption principle.

2. Let us analyze the wave field in the far zone. Applying the method of stationary phase [6] to (1.10), we obtain

$$\mathbf{u}_R(x, y, z) = \frac{i\cos\theta}{2\pi R}\sum_{n=1}^{2}\mathrm{Q}_n(\alpha_{1n}, \alpha_{2n})\,\varkappa_n e^{iR\varkappa_n}(1 + O(R^{-2})),\quad R\to\infty. \qquad (2.1)$$

A spherical coordinate system $x = R\cos\varphi\cdot\sin\theta$, $0 \leqslant \varphi \leqslant 2\pi$, $y = R\sin\varphi\cdot\sin\theta$, $\pi/2 \leqslant \theta \leqslant \pi$, $z = R\cos\theta$ is used here and $\alpha_{1n} = -\varkappa_n\sin\theta\cdot\cos\varphi$, $\alpha_{2n} = -\varkappa_n\sin\theta\cdot\sin\varphi$ are the stationary points. The first and second components of (2.1) describe the longitudinal and transverse waves, respectively.

An analytic representation of the solution corresponding to slightly damped surface waves is determined by residues at the real poles of the integrand and has the following form in the case of a homogeneous half-space

$$\mathbf{u}_r(x, y, z) = i\,\mathrm{res}\sum_{k=1}^{2}\left(\mathrm{Q}_k e^{\sigma_k z} + \mathrm{G}_k e^{-\sigma_k z}\right)\alpha\sqrt{\frac{2}{\alpha r\pi}}\, e^{i\left(\alpha r - \frac{\pi}{4}\right)}\Big|_{\alpha=\eta}(1 + O(r^{-1})),\ r\to\infty,$$

where η is a Rayleigh pole, and $r = \sqrt{x^2 + y^2}$. The relationships obtained permit determination of the amplitudes of the longitudinal, transverse, and Rayleigh waves in the far zone from the source as a function of the frequency, the depth at which the source lies, its linear dimensions, and also the form of the applied load and its distribution along the source dimensions.

3. A numerical analysis is performed on a digital computer for horizontal and vertical loads with the following parameters: a = 7 is the source linear dimension, h = 24, 48, 72 are depths at which it lies, $f = v_S/v_P = 0.2$ is a parameter characterizing the properties of the medium. Here v_S, $v_P = 1$ are the transverse and longitudinal wave velocities. The load is distributed along the source according to the law $f(\zeta) = k\zeta + b$. Here $\int_0^a f(\zeta)\,d\zeta = 1$, $f(a)/f(0) = \varepsilon$, $0 \leqslant \varepsilon \leqslant 1$, $k = b(\varepsilon - 1)/a$, $b = 2/(a(1 + \varepsilon))$.

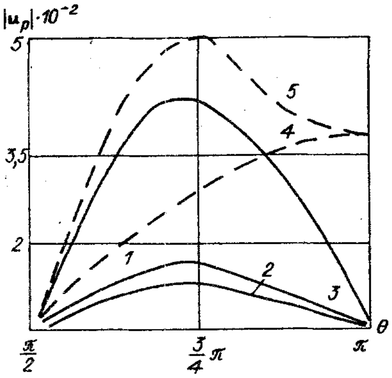

Fig. 1

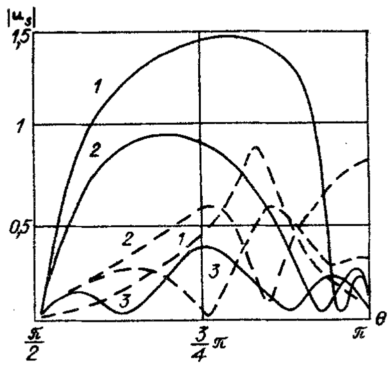

Fig. 2

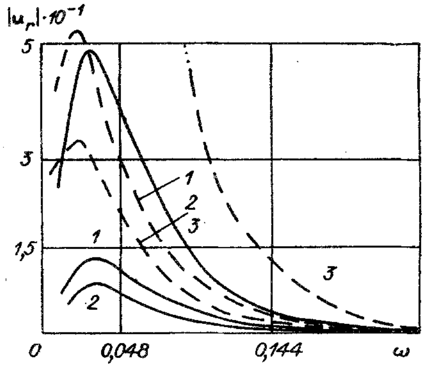

Fig. 3

Figure 1 shows shapes of the radiation directivity patterns for longitudinal waves for horizontal and vertical loads (lines 1-3 and 4, 5, respectively). The possibility of a non-uniform load distribution along the source dimensions was taken into account by the parameter $\varepsilon = 1, 0.5, 0$ (lines 1-3) for $\omega = 0.012$, $h = 24$. It is seen that as ε tends to zero the P-wave amplitude first diminishes and then grows abruptly. However, the tendency of this dependence is such that as the depth at which the source lies changes (for instance $h = 48$) only an increase in the longitudinal wave amplitude is observed as ε diminishes. Such changes are due to interaction between the waves emitted by the source and reflected from the half-space surface. Analogous dependences with respect to ε are characteristic even for the vertical load. Lines 4 and 5 correspond to $\omega_1 = 0.012$, $\omega_2 = 0.06$ for $\varepsilon = 1$, $h = 24$. The value $|\mathbf{u}_p|$ on line 4 diminishes three times.

It is seen that as the frequency increases lobes appear in the previous drop-shaped mode of the directivity pattern, in which the radiation maximum must now already be in a direction different from the strictly downward direction, and have a lower value. In the case of a horizontal load, an increase in the source vibrations frequency from ω_1 to ω_2 results in the radiation maximum being shifted in the directivity pattern towards lower values of the angle θ and to grow here.

Transverse wave directivity patterns for a vertical and horizontal load (solid and dashed lines, respectively) are represented in Fig. 2 for $\varepsilon = 1$, $j = 0.2$, $\omega = 0.012$. In each of the cases the depths $h = 24, 48, 72$ correspond to lines 1-3. The values of $|\mathbf{u}_s|$ on lines 1-3 diminished three times for the horizontal load. It is seen from Figs. 1 and 2 that the amplitude of the shifts in the transverse wave is greater than the amplitude of the shifts in the longitudinal wave by 1-2 orders on the average, depending on the kind of load. This is due to the fact that the source under consideration works in shear while, as a rule, longitudinal vibrations of significant amplitude excite sources operating in compression—expansion.

The dependence of the Rayleigh wave on the frequency is presented in Fig. 3. The solid lines correspond to a horizontal load, and the dashes to a vertical load. In both cases the lines 1-3 correspond to $\varepsilon = 1, 0.5, 0$ and $h = 24$.

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